II. *Animadverſions upon ſome Experiments relating to the Force of moving Bodies; with two new Experiments on the ſame Subject. By the Reverend Dr.* Deſaguliers, *F. R. S.*

IN a former Paper * I demonſtrated, by Reaſon and Experiments, that the *Momentum*, or Force of moving Bodies, is always proportionable to their Maſs multiplied into their Velocity; as it is the Opinion of the greateſt Part of the Mathematicians and Philoſophers. Now I come to conſider the Experiments, that have led ſome ingenious Men into an Error, in relation to this Propoſition. *Polenus* in his Book † gives an Account of his Experiments relating to this Matter, in theſe Words: "I took a Veſſel, that had in it congeal'd Tallow ſix Inches deep, and fix'd it to a "level Floor, in ſuch manner, that the Surface of "the Tallow, which was flat, ſhould every where be "equally diſtant from the Floor. I had cauſed to be "made two Balls of equal Bigneſs, the one of Lead, "the other of Braſs, the laſt of which was a little "hollow in the middle, that it might weigh but one "Pound, whilſt the other weigh'd two. Suſpending "theſe Balls from the Ceiling by Threads, in ſuch "manner, that the lighter Ball hung over the Surface "of the Tallow, from twice the Height that the heavier Ball did; I cut the Threads, and the Balls falling perpendicularly upon the Tallow, by their

* *Philoſoph. Tranſ.* No. 375. † De Caſtellis, pag. 56. No. 118.

" Fall made Pits in the Tallow, that were preciſely
" equal: the Ball of one Pound, from the Beginning
" of its Fall, till it came to reſt, going through a Space
" expreſs'd by the Number two, produc'd an Effect
" equal to that, which the two Pound Ball did pro-
" duce, in falling thro' a Space expreſs'd by the Num-
" ber one. It follows therefore, that we may look
" upon it as a ſettled Truth, That the active Forces
" (*vires vivas*) of falling Bodies are then equal, when
" their proper Weights are in a reciprocal *Ratio* of
" the Spaces, which the ſaid Bodies deſcribe by their
" Fall. And becauſe theſe Spaces are in the ſame *Ra-*
" *tio*, as the Squares of the Numbers expreſſing the Ve-
" locities; it appears by the Experiment, that the
" active Force (*vim vivam*) of the falling Body, is
" that which is made up of the Body itſelf, multiplied
" into the Space deſcribed in the Fall, or into the
" Square of the Number, that expreſſes the Velocity of
" the Body, at the end of the Motion. This Experi-
" ment I did not only make once, but ſeveral times,
" changing the Balls, the Diſtances, and the Body on
" which they fell; as for example, making uſe of Clay,
" or of ſoft Wax: and notwithſtanding theſe various
" ways of trying the Experiments, the Effects were
" conſtantly the ſame; which made me eaſily con-
" clude, that there was always the ſame Reaſon in Na-
" ture for this Phenomenon.

Thus far *Polenus*, whoſe Miſtake lies in this; that he eſtimates the Force of the Stroke of the falling Balls, by the Depth of the Impreſſion made in the Tallow, Clay, Wax, or any yielding Subſtance. But we muſt conſider, that when two Bodies move with equal Forces, but different Velocities, that, which moves the ſwifteſt, muſt make the deepeſt Impreſſion, whilſt the ſloweſt Body

Body communicates its Motion to the Clay round about, and therefore does not ſtrike in ſo deep as the ſwifter Body, which puts in Motion few Parts of the Clay, beſides thoſe that are before it, and which Parts have ſo much leſs Time to oppoſe this Body's Motion, as its Velocity is greater than the other's. To make this plainer, Let us ſuppoſe a Door half open, and moving very freely on its Hinges; if a Piſtol be fir'd againſt it, the Ball will go thro' the Door without moving it out of its Place; but if we take a large Weight of Lead, and throw it againſt the ſame Door, with the ſame Force as the Piſtol Bullet mov'd, the Door will be remov'd from its Poſition, and carried out of the Place on its Hinges by the Stroke; becauſe in the firſt Caſe, the Motion of the Ball is communicated but to a few Parts of the Door, and in the laſt it is diffus'd all over it. Nay, the Door will be mov'd by the Stroke, even tho' there ſhould be a prominent Part in the Lead, that ſhould be no bigger than a Piſtol-Bullet, in order to ſtrike the Door upon no more of its Surface, than the Bullet had done.

For illuſtrating this farther I contriv'd the following Experiment.

I caus'd a Machine to be made, as repreſented in *Pl.* I. *Fig.* 1. conſiſting of a Baſe of Wood AB, which could be ſet horizontal by means of three Screws, ſuch as SS: Upon this Board, or Baſe, there ſtood upright two parallel Boards, about four Inches wide, and four Inches aſunder, with the Elbow-piece EF ſliding behind one of them, ſo as to raiſe its upper End F to any Height deſired. Between theſe Boards, ſquare Frames of Wood GG *&c.* with Paper extended upon them, could ſlide in, to the Number of Six, in an horizontal

Poſition. Theſe Paper Diaphragms being thus plac'd, I ſuſpended an Ivory Ball of about one Inch and an half Diameter, weighing ſomething more than an Ounce and an half, by a ſhort Thread, under F, ſo that its Center of Gravity hung four foot over the firſt Diaphragm; then cutting the Thread, the Ball fell upon the Paper, and by its perpendicular Stroke broke thro' that Diaphragm, and the three next under it. Then putting ſo much Lead into the Ball abovemention'd, (which was made hollow for that Purpoſe) as to make it weigh twice as much as it did before; and bringing down F, to let it fall but from one foot, it broke thro' only two Diaphragms by its Fall. Making the Experiment ſeveral times with different Heights, but ſtill keeping the Proportion in Height of four to one, when the Balls were as one and two, the heavy and ſloweſt Ball broke thro' but half the Number of Papers. It happen'd indeed ſometimes, that there was ſome little difference, when the Papers were not equally ſtrong, or equally ſtretch'd, but the ſwifteſt Ball always broke through more Papers than the ſlow one.

Now tho' this Experiment does at firſt ſeem to confirm *Polenus*'s Theory; yet, when duly weigh'd, it proves no ſuch thing. For the lighter Ball does not break thro' more Papers, becauſe it has more Force, or a greater Quantity of Motion, but becauſe each Diaphragm has but half the time to reſiſt the Ball, that falls with a double Velocity, and therefore their Reſiſtance being as the time, as many more of them muſt be broken by the ſwift Ball, as by the ſlow one.

P. S. To all the Objectors, that allow the Force of moving Bodies, and their Quantity of Motion to be the ſame, what has been ſaid in this and my former Paper, ſeems to be a full Anſwer; but as there are now ſome Philoſophers, who diſtinguiſh that Force from the Quantity

Quantity of Motion, I am oblig'd to ſay ſomething more for the clearing up of that Point.

If I underſtand them right, they call *vis viva* a Force, whoſe Effect is ſenſible, as the Force of Gravity, when it accelerates Bodies in their Fall; and *vis mortua* a Force, which being deſtroy'd, produces no ſenſible Effect, as the Force of Gravity acting upon a Weight in one Scale of a Balance, when the Weight cannot deſcend by reaſon of a Counterpoiſe in the other Scale. But certainly no Man, that conſiders the thing attentively, would make that Diſtinction. However, ſince *Polenus* allows, that the Quantity of Motion in Bodies is as the Maſs multiplied into the Velocity (or MV); but ſays, that the Force, with which they act, which he diſtinguiſhes by the Name of *vis viva*, is as the Maſs multiplied into the Square of the Velocity, or MVV: I have made the following Experiment to ſhew his Notion to be inconſiſtent; tho all the *Phænomena* of unequal Weights applied to a *Statera*, ſo as to make an *Æquilibrium*, might ſerve for that Purpoſe, if it had not been objected, that the particular Conſtruction of the Machine hinder'd it from agreeing with the ſuppos'd Theorem, that the Force is as the Matter multiply'd into the Square of the Velocity.

EXPERIMENT.

Let two Balls, A and B, (*Pl.* I. *Fig.* 2.) be joyn'd by a String, which going thro' the ſmooth Hole C of an even Table, and under the Pulley P, ſuſpends a Weight W. It is plain, that upon letting go the Balls A and B, from the Places A and B, they will move towards C with the ſame Force, becauſe each of them will be drawn towards C by half the Force of the Weight W, whether the Balls be equal, or unequal.

1. The

1. The Balls being of two Ounces each (of Ivory), were, at the ſame inſtant of Time, let looſe from A and B, each diſtant twelve Inches from C, and both came to C at the ſame Time. Here the equal Forces will agree with the Product of the Maſſes into the Velocities, or into the Squares of the Velocities; becauſe $A \times 12 = B \times 12$, as well as $A \times 144$ is equal to $B \times 144$.

2. If A be taken of four Ounces Weight, and let go from D, or ſix Inches, whilſt B, ſtill equal to two, moves from 12 Inches; both Bodies will again meet at C: therefore here the equal Forces muſt be expreſs'd by the Maſſes into the Velocities, and not into their Squares; for tho' $A \times 6$ be equal to $B \times 12$ ($4 \times 6 = 2 \times 12$), $A \times 6 \times 6$, or 144 is but half of $B \times 12 \times 12$, or 288. Whereas if the Forces had been as *Polenus* affirms, B ſhould have been let looſe only from 8.4 Inches.

3. When A is ſix Ounces, it is let looſe only from E, or 4 Inches, to meet at C with B let looſe from 12; for then $A \times 4 = B \times 12$, whilſt $A \times 4 \times 4$, or 96, is three times leſs than $B \times 12 \times 12$, or 288. So that according to *Polenus*, B muſt have been let looſe from 7; but in that Caſe it comes ſooner to C than A.

N. B. The Weight W muſt be greater than the Weight of both Balls, leaſt the Friction of the Table ſhould ſpoil the Experiment.

III. *An*

Fig. 2.

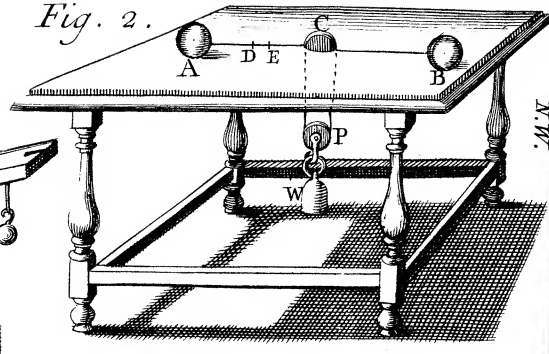

Fig. 1.

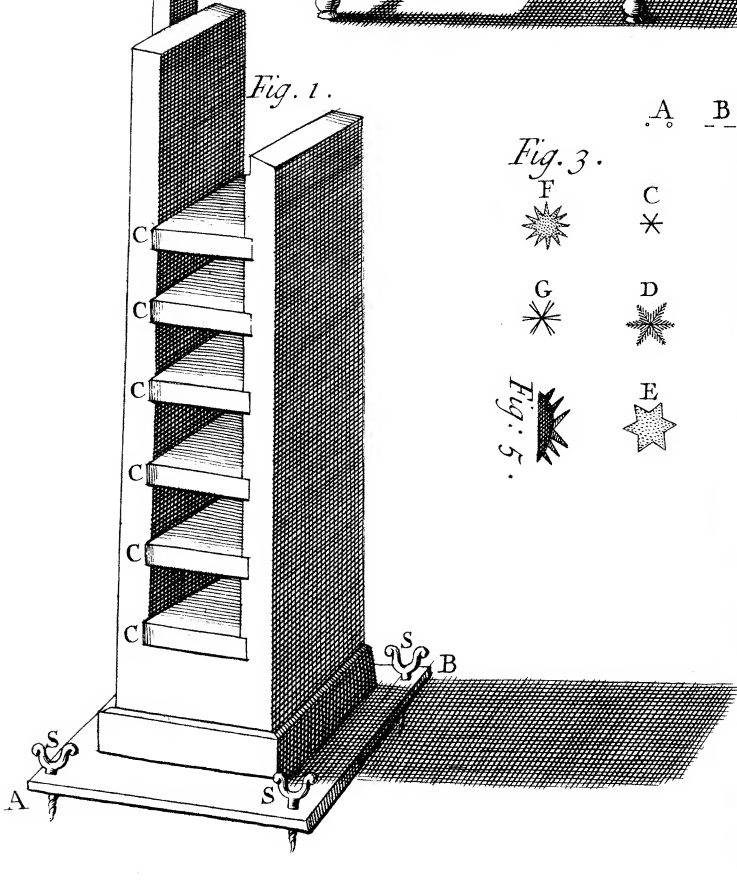

Fig. 3.

Fig. 5.

Fig. 4.